RESEARCH ARTICLE

Direct Displacement Control of Deformed Double Layer Dome

Shna Jabar Abdulkarim[1], Najmadeen Mohammed Saeed[1] & Hawkar Ali Haji[1]

[1]*Department of Civil Engineering, School of Engineering, University of Raparin, Sulaymaniyah, Kurdistan Region, F.R. Iraq*

Corresponding author's email: najmadeen_qasre@uor.edu.krd

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ABSTRACT

Space structures such as a double layer dome are light and active structural systems that are used for various structural applications, for instance, structural covers over large areas such as exhibition halls, stadiums, and concert halls. They are aesthetically pleasing in appearance, and serve the architectural requirement as well. The tolerances of structural shape under changing service conditions are important, and high appearance accuracy is required in some applications. Because of many reasons such as loadings, these type of structures may suffer from a noticeable deflection, which leads to a significant potential undesired appearance of the shape. In this situation, the displacements may need to be reduced or eliminated. In this study, by applying the shape adjustment technique, which depends on the linear force method, shape restoration is performed to the double layer dome model in three different cases corresponding to the directions of loadings as vertical, horizontal, and simultaneously vertical and horizontal were considered. The improvement of controlling nodal displacement can be achieved by using a rather simple and direct method, calculating necessary length of actuators by applying a single formulation. It is found that if the number of provided actuators are satisfactory, controlling of all the displaced joints could be performed and all the target joints could be restored to their original positions by a very small percentage of discrepancy as 0.5%, 1.8%, and 0.02% for the three considered cases, even if the controlled joints connection is not direct with the adjustable members. The technique of shape adjustment is very efficient for double layer dome model, and it can roughly eliminate the displacement of definite joints (exterior joints only) by simply changing the length of certain bars by $\mathbf{e_o}$ amount. The values of the total actuations are 874.95 mm, 246.25 mm, and 1150.8 mm in Cases 1, 2, and 3, respectively. Moreover, some of the members approve the better role in controlling the displaced joints as they are duplicated in all load cases, and they are sited in the inner layer of the double layer dome model.

Keywords: Force method, Actuation, Shape restoration, Displacement control, Double layer dome.

1. INTRODUCTION

In the past decades, there has been a growing number of structures using steel domes to cover large areas (Chen and Lui, 2005). Domes are one of the oldest magnificent structural systems. They consist of one or more layers of elements that are arched in all directions (Jayminkumar and Vahora, 2016). Domes are used to cover large areas such as exhibition halls, stadiums, and concert halls (Jayminkumar and Vahora, 2016) because they are lightweight and cultured assemblies that provide cost-effective solutions for covering large areas with their splendid aesthetic exterior. Devoid of disturbing columns in the interior with efficient shapes, dome covers the all-out volume and economy in terms of materials (Chen and Lui, 2005; Jayminkumar and Vahora, 2016). The geometry of the structure is an important aspect to prearrange the behavior, capacity utilization, and

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the heaviness of the structures (Chen and Lui, 2005).

Structural geometry is usually well-defined by determining nodal positions in both esthetical and functional aspects. The esthetical aspect includes egg-shaped space structures (Saeed et al., 2019), cable stayed bridge (Saeed and Kwan, 2017), and cable arch stayed bridges (Manguri et al., 2017; Saeed, 2019), whereas examples for functional aspect are antenna reflectors (You, 1997), large space antenna (Weeks, 1984), tetrahedral truss antenna reflector (Haftka and Adelman, 1985), space structures (Saeed et al., 2019), and cable stayed bridge (Saeed and Kwan, 2017). A high precision in those structural geometries with a high grade of accuracy is desired (Saeed et al., 2019; Saeed and Kwan, 2017; You, 1997).

However, the shape distortion of the structure is inevitable because of fatigue, imperfection in manufacturing, temperature deformation, unpredicted loading, and looseness in joints. As soon as the disfiguration shape is considered intolerable, the nodal positions require to be restored to its original shape. The technique of shape control/adjustment can be defined as reduction or even elimination of the structural deformation caused by external disturbances (Ziegler, 2005). Because of the capability of some members to elongate or shorten their length, the computational technique of shape restoration can be achieved (Manguri et al., 2017; Saeed et al., 2019; Saeed and Kwan, 2017; Saeed and Kwan, 2016; Shea et al., 2002; Xu and Luo, 2009; You, 1997). In addition, You (1997), as a straight issue, carried out the relation between the length of actuators and nodal displacements for not loaded prestressed structures. Saeed and Kwan (2016) provided a method to directly dominate nodal displacements by altering active members for structures distorted under loading.

Shape control/adjustment has been carried out on the different types of structures in order to eliminate the distortion of the shape geometry, for instance, the shape control of beam (Hadjigeorgiou et al., 2006; Yang and Ngoi, 2000; Yu et al., 2009), cable mesh antennas (Du et al., 2014; Mitsugi et al., 1990; Tanaka, 2011; Tanaka and Natori, 2006; Tanaka and Natori, 2004; Wang et al., 2013), intelligent structures (Wang et al., 1997), truss structures (Saeed and Kwan, 2016; Trak and Melosh, 1992), tensegrity structures (Shea et al., 2002), structural assemblies including complex elements "macro-elements," e.g., the pantographic element (Saeed and Kwan, 2016b), cable arch stayed bridges (Manguri et al., 2017; Saeed, 2019), and egg-shaped single layer space frames (Saeed et al., 2019).

In this study, the dome structure that is formed of two layers with interconnected elongated members is presented as a theoretical model that could undergo a great deformation under gravity loads or/and lateral loads. However, previously, the shape adjustment technique for space structure as the three dimensional egg-shaped single layer has been carried out by Saeed et al. (2019). Nonetheless, the displacement control was made only for vertical deformation that has been done by vertical loading only through adding the actuators as extra members before the stage of loading. It could be done during/after the process of construction to perform the process of adjustment, which means that the own members of the egg-shaped single layer space frame didn't participate in the adjustment process. Consequently, the focus of this paper is on the shape restoration by controlling deformation in all directions due to gravity loads or/and lateral loads by using the own members of the dome model as actuators.

The purpose of this paper is to use a direct relationship between bar length actuations and the nodal position/displacements for adjusting shape imperfection of the theoretical model of the double layer dome. This has been done using the method already derived by Saeed and Kwan (2014; 2016), using MATLAB program and validating by SAP2000 software. Besides, finding where the actuator should be placed led to the minimal amount of actuation, and it is possible to choose the optimal number of actuators.

2. GEOMETRY OF THE STRUCTURE AND LOADING

The structural assembly of the double layer dome and the properties of materials and loadings are represented as follows.

2.1. Geometry

The generation of the dome geometry is coded in MATLAB program as given in Fig. 1. The structure is analyzed through the force method. The theoretical model of double layer dome consists of 201 nodes and 760 members, which are arranged on the form of ribbed pattern to develop the dome model. The dome model has a diameter of 4000 mm with the ratio of span to height of 2. Moreover, the distance between both layers is indicated to be 100 mm, the ratio of its thickness to the diameter is 1/40, which is in the range of 1/30 to 1/60, as described by Chen and Lui (2005). The geometry is supported by 20 pinned supports along the perimeter, as shown in Fig. 2.

2.2. Properties of Materials

All members have been selected to have circular cross section with EA= 5.6549×10^{6}.

2.3. Loading

The structure has been loaded according to the three cases of loadings. In Case 1, the structure is loaded by 20 kN of gravity point loads on the exterior nodal positions as shown in Fig. 3. In Case 2, the structure is loaded by 5 kN point loads laterally on the exterior nodal positions as shown in Fig. 4. Finally, In Case 3, the structural geometry of the double layer dome model is loaded simultaneously by vertical and horizontal point loads of 20 kN and 5 kN, respectively, as shown in Fig. 5. Afterward, the geometry of the double layer dome model underwent a noticeable distortion in appearance, which is unacceptable. Hence, the technique of the shape restoration becomes necessary, as is stated in the following sections.

```
% Double Layer Ribbed Dome Geometry
Nr=20; Nc=6; %Nr: is number of meridians & Nc: is no. of circles
R=2000; Thick=100; %external radius (mm) & distance between two layers (mm)
Layer=2; j=0; w=0; Th=0; Alpha=0; %Layer: no. of layers, Th & Alpha: horizontal & vertical angle in degree
for Ly=1:Layer; for k=1:Nc-w; for i=1:Nr; j=j+1;
coor(j,1)= R*cosd(Alpha)*cosd(Th); coor(j,2)= R*cosd(Alpha)*sind(Th); coor(j,3)= R*sind(Alpha); Th=Th+360/Nr;
end; Alpha=Alpha+90/(Nc-1); end; Th=360/(2*Nr); Alpha=90/(2*(Nc-1));  w=w+1; R=R-Thick; end;
coor=coor([1:100,121:220,101],:);
for i=1:120;   bar(i,1)=i; bar(i,2)=i+1; end; for i=1:20:101; bar(i+19,2)=i;  end; %outer circle
for i=121:200; bar(i,1)=i; bar(i,2)=i+1; end; for i=121:20:181; bar(i+19,2)=i;end; %inner circle
j=1; %outer connection between circles
for i=201:280;  bar(i,1)=j; bar(i,2)=j+20; j=j+1; end; for i=281:300; bar(i,1)=j; bar(i,2)=201;j=j+1;end
%inner connection between circles
for i=301:380;  bar(i,1)=j; bar(i,2)=j+20; j=j+1; end; for i=381:400; bar(i,1)=j; bar(i,2)=201;j=j+1;end
j=101; k=1;  i=i+1; |  %  Connection between Two Layers
for h=1:4; for f=1:20; bar(i,1)=j; bar(i,2)=k;   i=i+1;
bar(i,1)=j; bar(i,2)=k+1; i=i+1; bar(i,1)=j; bar(i,2)=k+20;i=i+1;
bar(i,1)=j; bar(i,2)=k+21;i=i+1;j=j+1;k=k+1; end; end
for f=1:20; bar(i,1)=j; bar(i,2)=k;  i=i+1; bar(i,1)=j; bar(i,2)=k+1;i=i+1; j=j+1;k=k+1;end
bar(480,2)=1; bar(560,2)=21; bar(640,2)=41; bar(720,2)=61; bar(760,2)=81;
plot3([0 9], [0 6], [0 3], 'w'); hold on
for i=1:760;  j1=bar(i,1); j2=bar(i,2);
plot3([coor(j1,1) coor(j2,1)],  [coor(j1,2) coor(j2,2)],[coor(j1,3) coor(j2,3)],'-o','LineWidth',1,'Color','blue'); end;
```

Figure 1. MATLAB program code for geometry generation of the double layer dome model

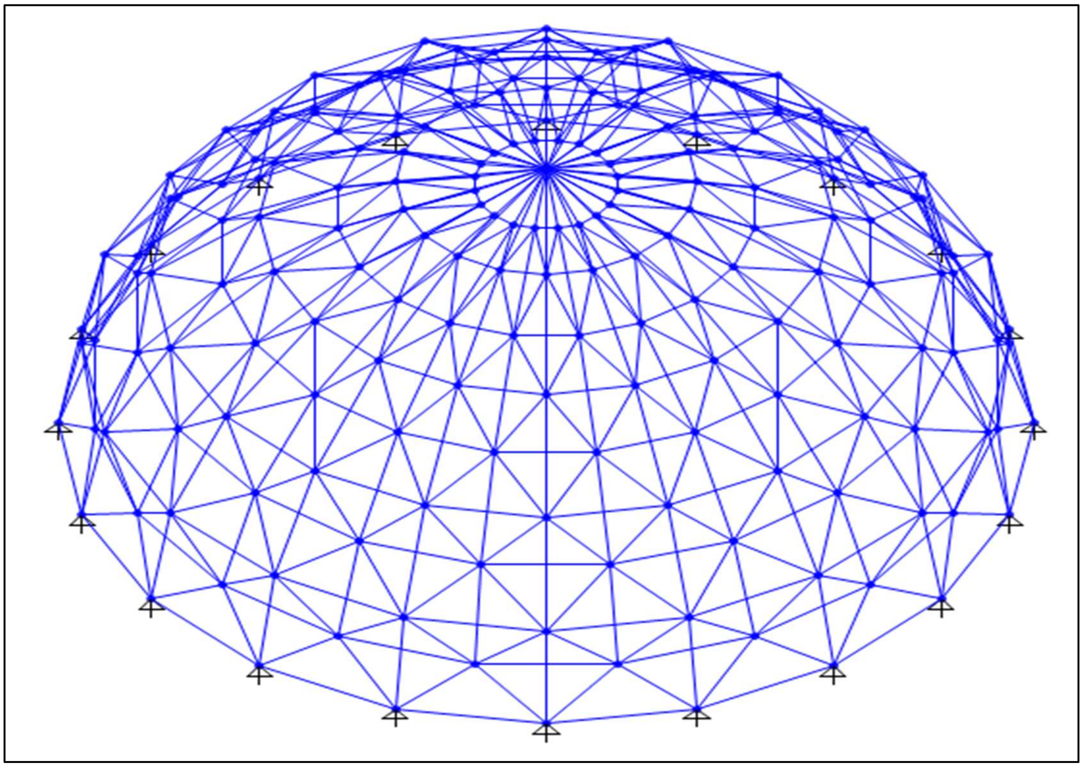

Figure 2. Geometry of double layer ribbed dome model

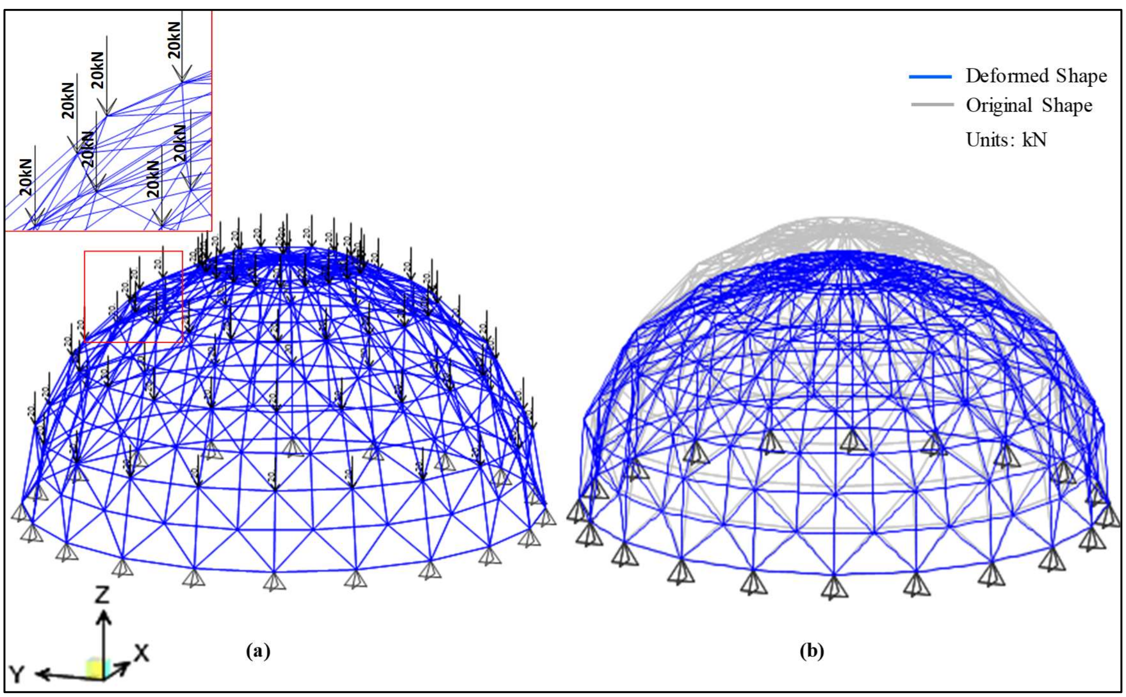

Figure 3. Double layer dome model (a) under vertical loading (b) deformed shape

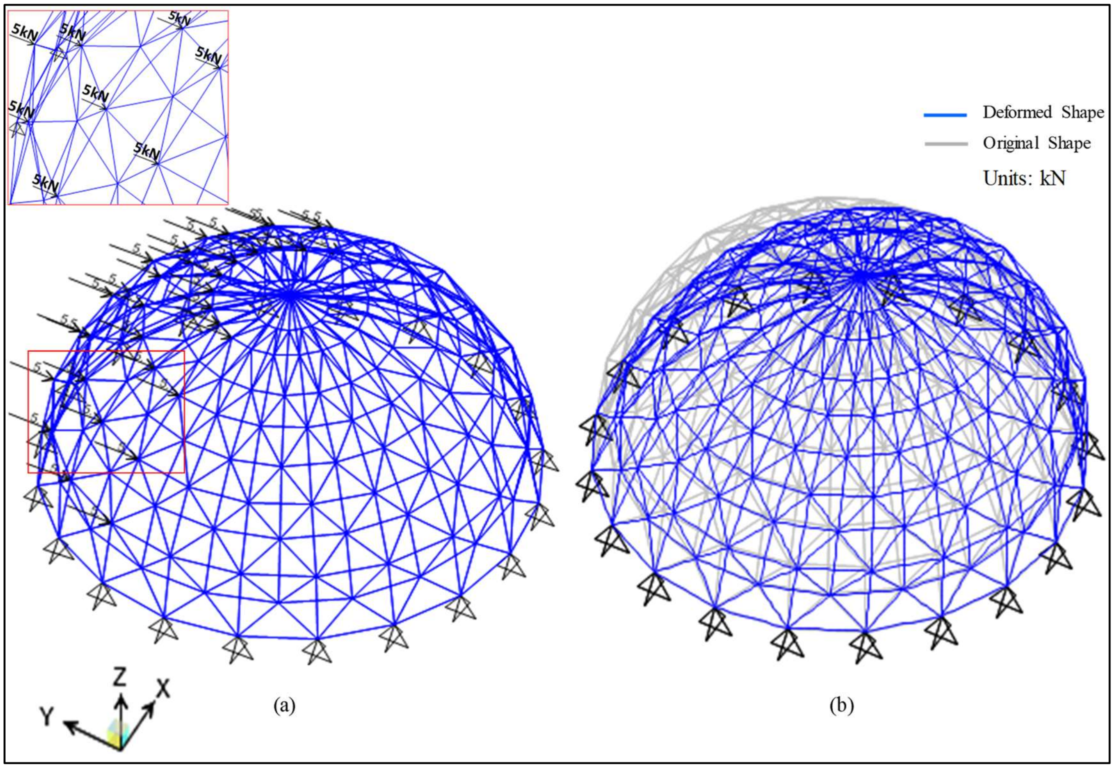

Figure 4. Double layer dome model (a) under horizontal loading (b) deformed shape

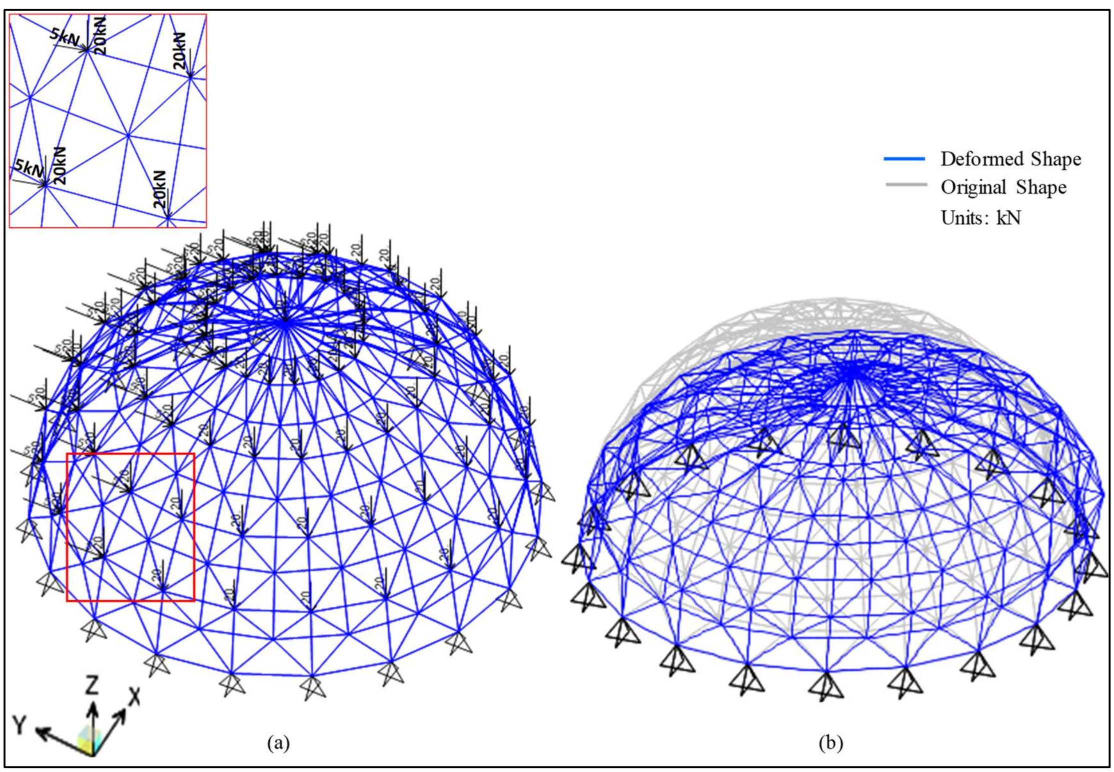

Figure 5. Double layer dome model (a) under vertical and horizontal loading simultaneously (b) deformed shape.

3. METHODOLOGY AND TECHNIQUE

The utilized technique of shape adjustment, based on the force method (Kwan, 1991; Pellegrino, 1993; Pellegrino et al., 1992), to allow "easy access" to the contributing parameters for the external displacements and internal deformation has been derived by Saeed and Kwan (2014; 2016). The current technique has been used for controlling of external deformation, internal force, and external deformation with internal force simultaneously, but in this paper, only the shape control is focused on. Eqn. 1 is the main equation of the technique, which is:

$$\mathrm{d} = \mathrm{Y}\boldsymbol{e_o} + \mathrm{d}_{\boldsymbol{P}} \quad (1)$$

where $\boldsymbol{Y} = \boldsymbol{B}^{+} - \boldsymbol{B}^{+}\boldsymbol{FS}(\boldsymbol{S}^{T}\boldsymbol{FS})^{-1}\boldsymbol{S}^{T}$, and $\boldsymbol{d}_{P} = [\boldsymbol{B}^{+}\boldsymbol{F} - \boldsymbol{B}^{+}\boldsymbol{FS}(\boldsymbol{S}^{T}\boldsymbol{FS})^{-1}\boldsymbol{S}^{T}\boldsymbol{F}]\boldsymbol{t}_{A}$ is the vector of nodal displacements of the structure due only to load, and **d** is the resultant nodal displacements after some elongation actuation $\mathbf{e_o}$ has been applied. The vector **d**, in whole or in part, can thus be used as the prescribed displacements, and eqn. 1 then provides the required corrective $\mathbf{e_o}$ to achieve that prescribed **d**, despite the effects of load in $\mathbf{d_P}$.

Clearly, **Y** is generally not a square matrix and need not even be of full rank. Furthermore, it is likely that only a few elements of **d** need to be controlled, and not all elements of $\mathbf{e_o}$ would typically be actuated. The system of equations and unknowns in eqn. 1 is thus normally likely to be only a small subset of the full set of equations. In view of all this, the solution for $\mathbf{e_o}$ is thus best obtained using the pseudoinverse of **Y**. For more straightforwardness and achieving elongation actuation $\mathbf{e_o}$ directly, eqn. 1 becomes:

$$\boldsymbol{e}_{o} = \boldsymbol{Y}^{+}[\boldsymbol{d} - \boldsymbol{d}_{P}] \quad (2)$$

4. RESULTS

In terms of applying different direction of loadings as vertical loadings, horizontal loadings, and both vertical and horizontal loadings simultaneously, three different situations are taken under consideration, namely, Case 1, Case 2, and Case 3, respectively. The target displacement of the model is restoring the surface shape to its original configuration (i.e., making all exterior joints have zero deflection in all directions **X**, **Y,** and **Z)** as shown in Column 5 in Tables 1-3. The number of total exterior joints is 81, which is determined from eqn. 2. The results of the displacement control for all the cases are presented in Tables 1-3, and the restored shapes of Cases 1, 2, and 3 are shown in Figs. 6, 7, and 8, respectively.

4.1. Shape Restoration under Vertical Loading Only (Case 1)

The double layer dome model is loaded by 20 kN of vertical point loads on the exterior joints, which are 81 joints, and the model confronted a great deformation as shown in Fig. 3. The X, Y, and Z directions of nodal displacement of joints ($\mathbf{d_P}$) after applying the loads are presented in Columns 2-4 of Table 1. Moreover, it can be noticed that the displacements in the direction of Z, which are matching with the direction of loadings are greater than displacements of both of X and Y directions.

Shape Restoration for Case 1: The desired target ($\mathbf{d_T}$) for restoring the displaced joints of exterior nodes (Column 1) for the double layer dome model are specified to be zero for all the directions as fixed in Column 5 of Table 1 by using the equation of adjustment (eqn. 2) (Saeed and Kwan, 2014, 2016). The shape adjustment technique has been performed and applied to the theoretical model via MATLAB program. A set of $\mathbf{e_o}$ is calculated to attain the desired target configuration, which is shown in Column 12 of Table 1. After applying this set of actuation $\mathbf{e_o}$ for the selected members (Column 14) of the deformed shape of the double layer dome model, the results of nodal displacements from the MATLAB program are presented in Columns 6-8 in Table 1. For the purpose of checking, another software SAP2000 is also used, and the offered results are showed in Columns 9-11. The results are very correlative with the achieved displacement results from the MATLAB program. All displacement of the post-adjustment in Columns 6-11 are very close to the desired target displacements in Column 5. The tiny discrepancy between the desire targets and the theoretical outcomes in this case were observed as shown in Fig. 6, which it is only 0.5% as a maximum discrepancy. The total amount of actuation by using 160 members of inner layer of the double layer dome model is 847.95 mm.

At the beginning stages of shape adjustment in the MATLAB program, all members of inner layer and all the interconnected members between inner and outer layers are chosen to participate in the shape restoration technique as actuators, which add up to 520 members. However, these stages provide the exact shape restoration as the desired target ($\mathbf{d_T}$), but using the 520 members is not economical and not applicable for the practical request. Therefore, depending on the **Y** in eqn. 2, that is totally governed by the geometry of the structure. The most active members, which have the larger coefficient values of **Y**, are chosen to perform the shape adjustment with fewer number of actuators. Finally, by after inspection as carried out by Saeed and Kwan (2016) through reselection of bars, the number of actuators are reduced to only 160 members. The results are very close to the desired target with the maximum discrepancy not exceeding 0.35 mm, as shown in Fig. 6.

For Case 1, where the direction of loadings is vertical, the active members that perform all the role in the shape adjustment technique are those members that are positioned in the inner layer of the double layer dome model. Besides, the interconnected members do not confirm a significant function for the shape restoration technique. In addition, the number of actuators still can be reduced, but as a consequence, it increases the range of maximum discrepancy.

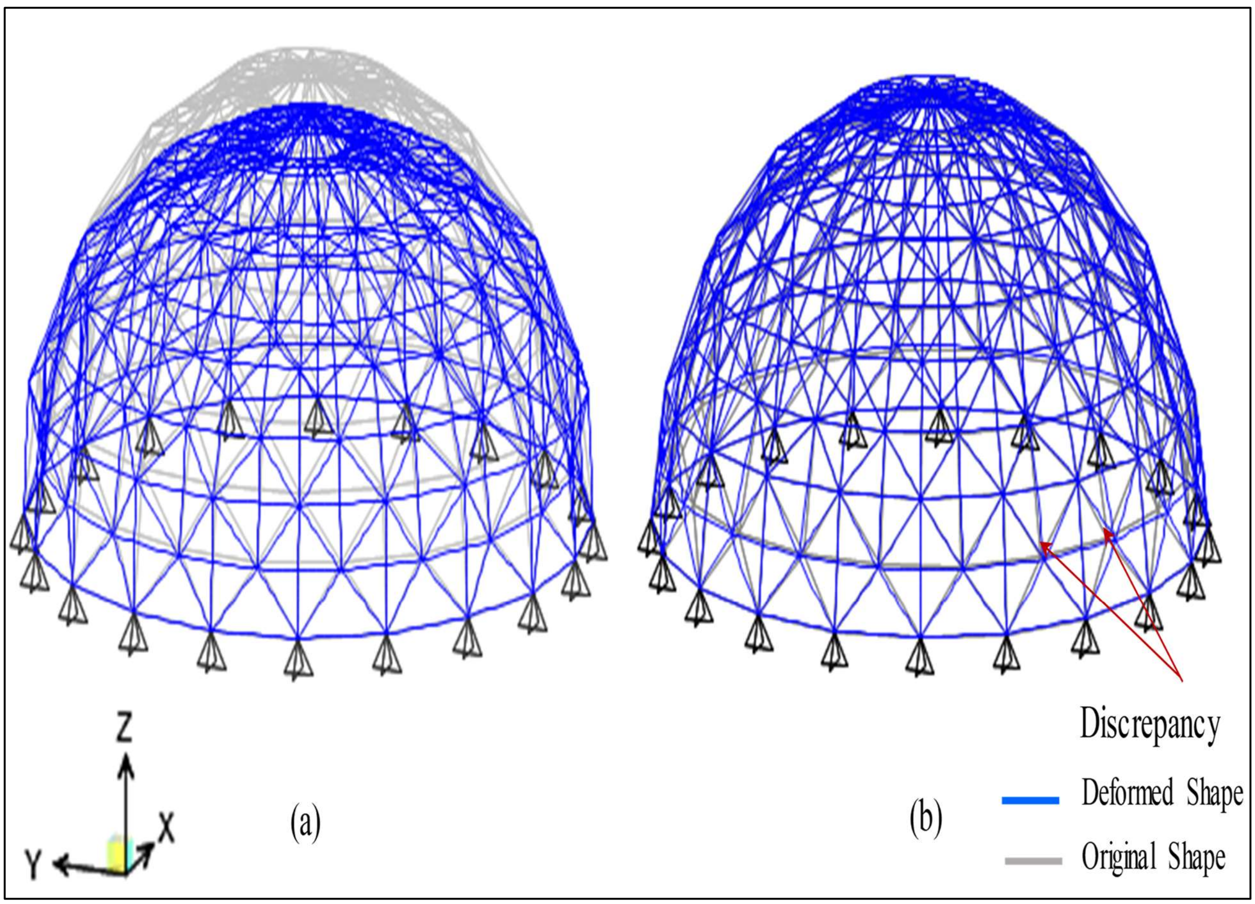

Figure 6. Double layer dome model under vertical loading (a) Pre-adjustment (b) Post-adjustment

Table 1: Shape restoration of the Dome Model under vertical loading only

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Joints	d_P (mm) MATLAB & SAP2000			d_T (mm)	d (mm) MATLAB			d (mm) SAP2000			Single Bar e_o (mm)	Twenty Bar e_o (mm)	Actuators
	X	Y	Z	X, Y, & Z	X	Y	Z	X	Y	Z			
1-20	0	0	0								9.790	195.798	101 - 120
21,31*	6.79	0.00	-4.00	0	0.00	0.00	0.00	0.02	0.00	0.00	-4.453	-89.068	121 - 140
22,30*	6.45	2.10	-4.00	0	0.00	0.00	0.00	0.02	0.01	0.00	-0.395	-7.894	141 - 160
23,29*	5.49	3.99	-4.00	0	0.00	0.00	0.00	0.02	0.01	0.00	1.989	39.770	181 - 200
24,28*	3.99	5.49	-4.00	0	0.00	0.00	0.00	0.01	0.02	0.00	14.683	293.668	301 - 320
25,27*	2.10	6.45	-4.00	0	0.00	0.00	0.00	0.01	0.02	0.00	6.327	126.546	321 - 340
26	0.00	6.79	-4.00	0	0.00	0.00	0.00	0.00	0.02	0.00	3.632	72.640	341 - 360
32,40*	-6.45	6.45	-4.00	0	0.00	0.00	0.00	-	-	0.00	1.128	22.564	361 - 380
33,39*	-5.49	5.49	-4.00	0	0.00	0.00	0.00	-	-	0.00			
34,38*	-3.99	3.99	-4.00	0	0.00	0.00	0.00	-	-	0.00			
35,37*	-2.10	2.10	-4.00	0	0.00	0.00	0.00	-	-	0.00			
36	0.00	0.00	-4.00	0	0.00	0.00	0.00	0.00	-	0.00			
41,51*	5.04	0.00	-7.55	0	0.05	0.00	-0.05	0.05	0.00	-			
42,50*	4.79	1.56	-7.55	0	0.05	0.02	-0.05	0.05	0.02	-			
43,49*	4.08	2.96	-7.55	0	0.04	0.03	-0.05	0.04	0.03	-			
44,48*	2.96	4.08	-7.55	0	0.03	0.04	-0.05	0.03	0.04	-			
45,47*	1.56	4.79	-7.55	0	0.02	0.05	-0.05	0.02	0.05	-			
46	0.00	5.04	-7.55	0	0.00	0.05	-0.05	0.00	0.05	-			
52,60*	-4.79	4.79	-7.55	0	-0.05	-0.02	-0.05	-	-	-			
53,59*	-4.08	4.08	-7.55	0	-0.04	-0.03	-0.05	-	-	-			
54,58*	-2.96	2.96	-7.55	0	-0.03	-0.04	-0.05	-	-	-			
55,57*	-1.56	1.56	-7.55	0	-0.02	-0.05	-0.05	-	-	-			
56	0.00	0.00	-7.55	0	0.00	-0.05	-0.05	0.00	-	-			
61,71*	-0.57	0.00	-15.90	0	-0.34	0.00	0.16	-	0.00	0.14			
62,70*	-0.55	-0.18	-15.90	0	-0.33	-0.11	0.16	-	-	0.14			
63,69*	-0.46	-0.34	-15.90	0	-0.28	-0.20	0.16	-	-	0.14			
64,68*	-0.34	-0.46	-15.90	0	-0.20	-0.28	0.16	-	-	0.14			
65,67*	-0.18	-0.55	-15.90	0	-0.11	-0.33	0.16	-	-	0.14			
66	0.00	-0.57	-15.90	0	0.00	-0.34	0.16	0.00	-	0.14			
72,80*	0.55	-0.55	-15.90	0	0.33	0.11	0.16	0.33	0.11	0.14			
73,79*	0.46	-0.46	-15.90	0	0.28	0.20	0.16	0.28	0.20	0.14			
74,78*	0.34	-0.34	-15.90	0	0.20	0.28	0.16	0.20	0.28	0.14			
75,77*	0.18	-0.18	-15.90	0	0.11	0.33	0.16	0.11	0.33	0.14			
76	0.00	0.00	-15.90	0	0.00	0.34	0.16	0.00	0.35	0.14			
81,91*	-2.45	0.00	-24.16	0	0.31	0.00	-0.12	0.31	0.00	-			
82,90*	-2.33	-0.76	-24.16	0	0.29	0.10	-0.12	0.29	0.10	-			
83,89*	-1.98	-1.44	-24.16	0	0.25	0.18	-0.12	0.25	0.18	-			
84,88*	-1.44	-1.98	-24.16	0	0.18	0.25	-0.12	0.18	0.25	-			
85,87*	-0.76	-2.33	-24.16	0	0.10	0.29	-0.12	0.10	0.29	-			
86	0.00	-2.45	-24.16	0	0.00	0.31	-0.12	0.00	0.31	-			
92,100*	2.33	-2.33	-24.16	0	-0.29	-0.10	-0.12	-	-	-			
93,99*	1.98	-1.98	-24.16	0	-0.25	-0.18	-0.12	-	-	-			
94,98*	1.44	-1.44	-24.16	0	-0.18	-0.25	-0.12	-	-	-			
95,97*	0.76	-0.76	-24.16	0	-0.10	-0.29	-0.12	-	-	-			
96	0.00	0.00	-24.16	0	0.00	-0.31	-0.12	0.00	-	-			
201	0.00	0.00	-17.35	0	0.00	0.00	0.35	0.00	0.00	0.33			
											Total actuation (mm)	**847.95**	

*These numbers have the opposite sign in x-direction.

4.2. Shape Restoration under Horizontal Loading Only (Case 2)

In this case, the double layer dome model is laterally loaded in Y direction by 5 kN of point loads on the half part of the exterior joints, which are 36 joints, and the model displayed a noticeable deformation, as shown in Fig. 4. The displacements of X, Y, and Z directions of joints ($\mathbf{d_P}$) after affecting by the loads are presented in Table 2 in Columns 2-4. Likewise, it can be noticed that the displacements in the Y direction, which are parallel to the load direction, have the greatest value compared with the other directions for all the joints.

Shape Restoration for Case 2: Same as the Case 1, the desired target ($\mathbf{d_T}$) that should be restored for the displaced joints of exterior nodes (Column 1) for the double layer dome model is specified to be zero for all the directions X, Y, and Z, as shown in Column 5 of Table 2. From eqn. 2, a group variety of $\mathbf{e_o}$ is determined to achieve the desired target shape, which is demonstrated in Columns 12 and 13 of Table 2. After stratifying this set of length alteration ($\mathbf{e_o}$) for the nominated members in Column 14, the theoretical result $\mathbf{d}$ (Columns 6-11) is very close to the desired target (Column 5). The results of this case are also validated by the SAP2000 software, which are very correlative with the achieved displacement results from the MATLAB program as shown in Columns 9-11. The amount of 246.25 mm is the total actuation for 80 members of inner layer and interconnected members of the double layer dome model. Correspondingly, the difference between the desired targets and the theoretical outcomes in this case, as the maximum discrepancy is 1.8% roughly.

Similar to the Case 1, all of the 520 members (inner layer and interconnected members) are specified to effort as the actuators and gave the amount of $\mathbf{d}$ as the exact value of desired target of the controlled displacement. In the next stages, depending on the reselection of bars (Saeed and Kwan, 2016), the number of actuators are decreased to 80. These number of members as the actuators provide an acceptable outcome to the desired target within the maximum discrepancy equal to 0.56 mm (1.8%), as shown in Fig. 7. In this Case, where the direction of loadings is only within the horizontal Y direction, the active members that perform the shape adjustment technique are located in the inner layer and the interconnected members of the double layer dome model. Moreover, the interconnected members play an important role in the shape restoration technique. If the actuators are compared between both Cases 1 and 2 in Column 14 of Tables 1 and 2, there are 26 members that are duplicated in the adjustment process that are located in the inner layer of the double layer dome model. Furthermore, the number of actuators can be further reduced, but it will increase the range of maximum discrepancy.

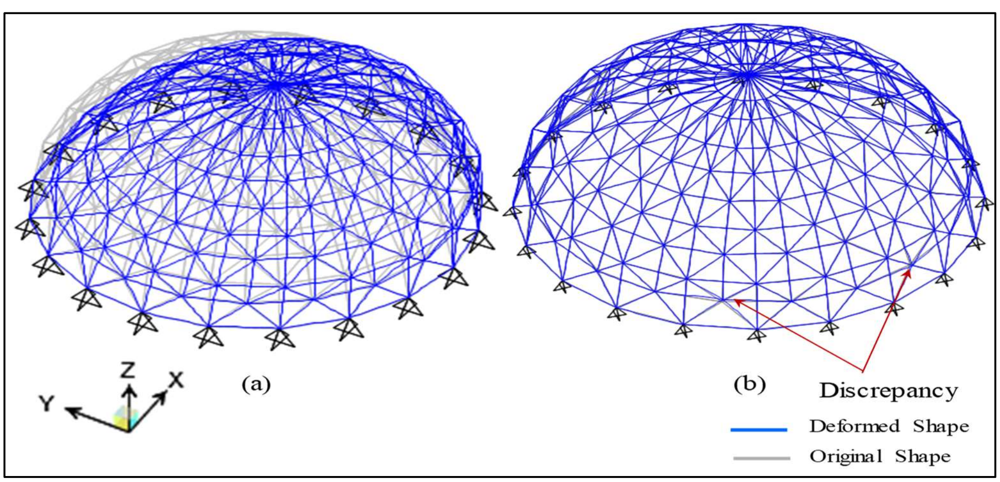

Figure 7. Double layer dome model under horizontal loading (a) Pre-adjustment (b) Post-adjustment

4.3. Shape Restoration under Vertical and Horizontal Loading Simultaneously (Case 3)

In this case, the double layer dome model is laterally loaded in Y direction by 5 kN and also vertically loaded in Z direction by 20 kN simultaneously. The model is exposed to an observable distortion, as shown in Fig. 5. The considerable situation is that some of the displacements of X, Y, and Z directions of the joints due to the loadings are considered to be unacceptable. Therefore, the geometry of the model should be restored by using some actuators that should be imbedded to the most active bars, as presented in Columns 2-4 of Table 3. Compatibly, the double layer dome model in Case 3 behaves like the model in Case 1 corresponding to the Z direction, whereas it behaves like the model in Case 2 for X and Y directions. Nonetheless, all the values of the displacements are less than the corresponding displacement of the two former cases.

Table 2: Shape restoration of the Dome Model under horizontal loading only

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Joints	d_P (mm)			d_T (mm)	d (mm)						Single Bar e_o (mm)	Both Bar e_o (mm)	Actuators
	MATLAB & SAP2000				MATLAB			SAP2000					
	X	Y	Z	X, Y, & Z	X	Y	Z	X	Y	Z			
1-20	0	0	0								3.32	6.63	121,129
21,31*	0.53	-2.38	0.17	0	0.00	-0.08	-0.06	0.00	-0.09	-0.06	4.55	9.10	123,127
22,30*	-0.27	-2.86	0.07	0	0.10	0.14	-0.13	0.09	0.13	-0.14	1.71	1.71	125
23,29*	-0.98	-3.70	-0.08	0	-0.01	0.05	0.16	-0.02	0.04	0.15	-2.23	-2.23	135
24,28*	-1.15	-4.72	-0.21	0	-0.19	-0.06	0.19	-0.19	-0.06	0.19	2.18	4.37	144,146
25,27*	-0.75	-5.51	-0.29	0	0.04	0.23	-0.01	0.04	0.23	-0.01	3.76	7.51	161,169
26	0.00	-5.80	-0.31	0	0.00	-0.20	0.23	0.00	-0.20	0.23	-2.34	-4.67	173,177
32,40*	-0.74	-2.38	0.08	0	-0.15	0.01	-0.18	-0.16	0.01	-0.17	-3.22	-6.44	303,308
33,39*	-0.72	-2.49	-0.05	0	-0.04	-0.09	-0.05	-0.05	-0.11	-0.04	-1.65	-3.30	305,306
34,38*	-0.60	-2.70	-0.14	0	-0.05	0.04	-0.22	-0.05	0.05	-0.23	3.46	6.93	314,317
35,37*	-0.35	-2.91	-0.19	0	0.15	-0.08	-0.15	0.15	-0.08	-0.15	-0.96	-1.91	315,316
36	0.00	-2.99	-0.21	0	0.00	0.07	-0.25	0.00	0.07	-0.25	1.87	3.74	333,338
41,51*	0.57	-4.45	0.44	0	0.06	-0.22	-0.03	0.06	-0.22	-0.03	1.91	3.82	334,337
42,50*	-0.18	-4.99	0.04	0	-0.21	0.19	0.15	-0.21	0.19	0.15	2.15	4.29	355,356
43,49*	-0.85	-5.86	-0.47	0	0.04	-0.04	-0.03	0.04	-0.04	-0.03	-5.22	-10.43	361,370
44,48*	-1.04	-6.89	-0.91	0	0.15	-0.09	-0.02	0.15	-0.09	-0.02	3.32	6.65	375,376
45,47*	-0.69	-7.68	-1.18	0	-0.08	-0.28	0.14	-0.08	-0.28	0.14	-4.59	-9.19	402,437
46	0.00	-7.98	-1.27	0	0.00	-0.03	-0.23	0.00	-0.03	-0.23	-8.37	-16.73	406,433
52,60*	-0.85	-4.39	0.51	0	-0.17	0.12	-0.07	-0.17	0.11	-0.07	2.41	4.82	407,436
53,59*	-0.86	-4.50	0.46	0	0.13	0.01	0.14	0.13	0.01	0.14	1.72	3.43	417,422
54,58*	-0.72	-4.71	0.44	0	-0.23	0.01	-0.06	-0.23	0.01	-0.06	-2.59	-5.17	441,480
55,57*	-0.42	-4.93	0.44	0	0.10	-0.05	0.01	0.10	-0.05	0.01	9.41	18.81	446,473
56	0.00	-5.02	0.44	0	0.00	-0.37	-0.24	0.00	-0.37	-0.24	3.90	7.79	448,475
61,71*	0.26	-6.02	0.43	0	-0.23	0.20	0.11	-0.23	0.20	0.11	3.72	7.43	484,519
62,70*	-0.18	-6.51	-0.13	0	-0.15	0.10	0.07	-0.15	0.10	0.07	1.19	2.38	488,515
63,69*	-0.50	-7.11	-0.70	0	-0.48	-0.02	-0.07	-0.48	-0.02	-0.07	-2.96	-5.92	489,510
64,68*	-0.57	-7.73	-1.18	0	0.13	0.23	0.01	0.13	0.23	0.01	2.71	5.41	492,511
65,67*	-0.37	-8.18	-1.49	0	0.01	0.00	-0.17	0.01	0.00	-0.17	-3.13	-6.27	521,560
66	0.00	-8.35	-1.60	0	0.00	0.15	0.44	0.00	0.15	0.44	1.61	3.22	523,558
72,80*	-0.56	-5.89	0.81	0	-0.12	0.31	0.09	-0.12	0.31	0.09	-1.60	-3.20	527,556
73,79*	-0.69	-5.98	1.10	0	0.05	0.43	-0.08	0.05	0.42	-0.08	-3.82	-7.63	528,555
74,78*	-0.63	-6.20	1.32	0	-0.34	0.00	0.15	-0.34	-0.01	0.15	-1.37	-2.74	530,549
75,77*	-0.38	-6.41	1.46	0	-0.18	0.14	-0.11	-0.18	0.14	-0.11	2.62	5.25	531,552
76	0.00	-6.50	1.52	0	0.00	-0.35	0.07	0.00	-0.35	0.07	2.24	4.48	561,598
81,91*	0.21	-7.12	0.58	0	0.11	-0.26	0.21	0.11	-0.26	0.21	3.45	6.90	563,600

82,90*	0.09	-7.39	0.03	0	0.04	-0.18	-0.56	0.04	-0.18	-0.56	3.93	7.86	564,599
83,89*	0.00	-7.63	-0.43	0	-0.11	-0.18	0.20	-0.11	-0.18	0.20	-1.53	-3.05	567,596
84,88*	-0.03	-7.82	-0.78	0	-0.08	-0.13	0.17	-0.08	-0.13	0.17	-2.73	-5.46	601,640
85,87*	-0.03	-7.94	-1.01	0	-0.06	-0.19	-0.10	-0.06	-0.19	-0.10	3.10	6.21	606,633
86	0.00	-7.98	-1.08	0	0.00	-0.23	-0.22	0.00	-0.23	-0.22	-2.95	-5.89	609,630
92,100*	-0.28	-6.99	1.14	0	0.02	-0.19	0.16	0.02	-0.19	0.16	3.62	7.25	645,674
93,99*	-0.31	-6.96	1.65	0	0.14	-0.14	0.03	0.14	-0.14	0.03			
94,98*	-0.27	-7.00	2.07	0	0.07	0.02	0.10	0.07	0.02	0.10			
95,97*	-0.16	-7.05	2.33	0	0.02	0.21	-0.01	0.02	0.21	-0.01			
96	0.00	-7.08	2.42	0	0.00	0.28	0.11	0.00	0.28	0.11			
201	0.00	-7.09	1.29	0	0.00	0.30	0.04	0.00	0.30	0.04			
Total actuation (mm)												246.25	

Shape Restoration for Case 3: As mentioned in the two earlier cases, the desired target ($\mathbf{d_T}$) for exterior joints (Column 1) of the double layer dome model is indicated to be zero for the three dimensional directions as presented in Column 5 of Table 3. The determined set of $\mathbf{e_o}$ due to applying eqn. 2 has been calculated and managed in Columns 12 and 13 in Table 3 after applying this set of member actuation ($\mathbf{e_o}$) for the selected members of the theoretical model (Column 14). The outcomes of restored displacement $\mathbf{d}$ by SAP2000 software, which are very correlative with the achieved displacement by MATLAB program, are presented in Columns 6-11; correspondingly, the results are very close to the desired target (Column 5), with the maximum discrepancy of 0.18 mm (0.02%). The total amount of actuation by requesting 203 members of inner layer and interconnected members of the double layer dome model is only 1150.8 mm.

Similarly, for all the cases at the establishment stages, all the members except the exterior members, which are 520 members, are chosen to contribute in the process of restoration as actuators. However, the outcome $\mathbf{d}$ of utilizing all these members provides the exact shape restoration as the desired target ($\mathbf{d_T}$), as in both previous cases. However, using the 520 members will be costly and not applicable for hands-on application. Therefore, depending on the maximum coefficient of $\mathbf{Y}$ in eqn. 2 (Saeed and Kwan, 2016), the most active members are chosen to perform the shape adjustment technique, as shown in Fig. 8.

For Case 3, where the direction of loadings is vertical and horizontal simultaneously, the active members that accomplish the performance in the shape adjustment technique are both of the interconnected members and inner layer members of the dome model. In addition, the duplicated members between Cases 1 and 2 are kept to participate in the shape restoration technique. Furthermore, the number of elongation members can be reduced, but this reduction will increase the range of maximum discrepancy and shows grater variance between the desire target and obtained nodal displacement.

Table 3: Shape restoration of the Dome Model under vertical and horizontal loading simultaneously

1	2	3	4	5	6	7	8	9	10	11	12	13	14			
Joints	d_P (mm)			d_T (mm)	d (mm)						Single Bar e_o (mm)	Both Bar e_o (mm)	Actuators	Continues Column No. 12	Continues of Column No. 13	Continues of Column No. 14
	MATLAB & SAP2000				MATLAB			SAP2000								
	X	Y	Z	X, Y, & Z	X	Y	Z	X	Y	Z						
1-20	0	0	0								14.86	29.73	103,107	5.96	11.93	354,357
21,31*	7.32	-2.38	-3.83	0	-0.04	-0.10	-0.01	-0.04	-0.10	-0.01	-2.32	-4.65	104,106	15.44	30.87	355,356
22,30*	6.18	-0.76	-3.93	0	0.02	-0.01	0.01	0.06	-0.01	0.04	-10.20	-10.20	115	3.79	7.58	361,370
23,29*	4.51	0.28	-4.08	0	-0.02	-0.03	0.06	0.01	0.00	0.09	-6.32	-12.63	121,129	-1.04	-2.08	362,369
24,28*	2.84	0.77	-4.21	0	0.02	0.00	-0.01	0.02	-0.01	-0.02	-9.09	-18.19	122,128	2.78	5.56	363,368

25,27*	1.35	0.94	-4.28	0	-0.02	0.00	-0.03	-0.01	0.01	-0.03	-5.77	-11.54	123,127	4.23	8.46	365,366
26	0.00	0.98	-4.31	0	0.00	0.07	-0.10	0.00	0.09	-0.09	-0.49	-0.49	125	3.58	7.15	373,378
32,40*	-7.19	-4.48	-3.91	0	-0.01	-0.06	0.01	0.01	-0.05	0.00	3.59	7.18	131,139	1.14	2.28	374,377
33,39*	-6.21	-6.48	-4.05	0	-0.01	0.01	-0.02	-0.03	0.00	0.01	-3.72	-7.45	132,138	-1.72	-3.43	375,376
34,38*	-4.59	-8.19	-4.14	0	0.00	0.00	-0.01	0.01	-0.03	-0.01	-11.64	-23.28	133,137	8.52	17.04	401,438
35,37*	-2.45	-9.36	-4.19	0	0.00	0.00	0.00	-0.02	-0.02	0.02	-10.65	-21.30	134,136	8.31	16.62	402,437
36	0.00	-9.78	-4.20	0	0.00	0.01	0.01	0.00	-0.08	0.08	-7.67	-7.67	135	14.13	28.25	405,434
41,51*	5.61	-4.45	-7.11	0	0.08	0.07	-0.02	0.07	0.07	-0.02	-3.30	-6.60	143,147	17.81	35.62	406,433
42,50*	4.61	-3.43	-7.51	0	-0.02	0.00	-0.03	-0.03	0.00	-0.03	-5.39	-10.78	144,146	9.58	19.15	409,430
43,49*	3.22	-2.90	-8.02	0	-0.01	0.06	-0.01	-0.01	0.06	-0.01	-3.05	-6.09	153,157	-2.43	-4.87	416,427
44,48*	1.92	-2.81	-8.46	0	0.05	0.01	-0.03	0.05	0.01	-0.03	-4.12	-8.23	154,156	10.10	20.20	417,422
45,47*	0.87	-2.89	-8.73	0	-0.01	0.01	0.01	-0.01	0.01	0.00	-5.11	-5.11	155	5.64	11.28	447,476
46	0.00	-2.94	-8.83	0	0.00	-0.05	0.08	0.00	-0.05	0.07	2.26	4.52	161,169	11.73	23.46	449,470
52,60*	-5.64	-5.95	-7.04	0	0.00	0.06	-0.03	0.00	0.06	-0.03	1.44	2.89	162,168	14.53	29.07	451,472
53,59*	-4.94	-7.46	-7.09	0	0.03	-0.03	0.01	0.03	-0.04	0.02	1.15	2.30	163,167	-7.79	-15.57	453,466
54,58*	-3.68	-8.79	-7.11	0	0.01	0.01	0.01	0.00	0.00	0.02	3.82	7.64	170,180	4.35	8.70	454,465
55,57*	-1.98	-9.72	-7.11	0	-0.01	0.00	-0.01	-0.01	-0.02	0.00	1.85	3.70	171,179	22.83	45.67	457,462
56	0.00	-10.06	-7.11	0	0.00	-0.02	-0.02	0.00	-0.02	-0.03	2.76	5.52	172,178	19.01	38.01	458,461
61,71*	-0.31	-6.02	-15.46	0	-0.03	0.00	0.05	-0.03	0.00	0.05	1.92	3.83	173,177	-1.84	-3.67	484,519
62,70*	-0.72	-6.69	-16.02	0	0.05	-0.05	-0.04	0.05	-0.06	-0.04	3.10	6.21	181,189	2.05	4.10	492,511
63,69*	-0.97	-7.45	-16.60	0	-0.08	0.05	0.03	-0.08	0.05	0.02	2.95	5.90	182,188	3.24	6.49	496,507
64,68*	-0.91	-8.19	-17.08	0	-0.06	-0.01	0.04	-0.06	-0.01	0.04	2.08	4.15	183,187	2.40	4.81	500,503
65,67*	-0.55	-8.73	-17.39	0	-0.07	-0.01	0.02	-0.07	-0.01	0.02	2.06	4.12	184,186	4.27	8.54	527,556
66	0.00	-8.93	-17.50	0	0.00	0.01	-0.02	0.00	0.01	-0.02	1.62	1.62	185	6.78	13.57	531,552
72,80*	-0.01	-5.71	-15.08	0	-0.02	-0.01	-0.01	-0.02	-0.01	-0.01	2.39	4.78	190,200	6.93	13.85	535,548
73,79*	-0.22	-5.65	-14.80	0	-0.02	0.00	-0.02	-0.02	-0.01	-0.02	2.67	5.34	191,199	2.21	4.42	536,547
74,78*	-0.29	-5.74	-14.58	0	0.01	-0.02	-0.01	0.01	-0.02	-0.01	4.64	9.28	303,308	-1.76	-3.51	540,543
75,77*	-0.20	-5.87	-14.43	0	0.00	0.01	0.01	0.01	0.01	0.01	17.36	34.72	304,307	7.85	15.70	575,588
76	0.00	-5.93	-14.38	0	0.00	0.03	0.02	0.00	0.04	0.02	-0.51	-1.02	305,306	8.14	16.29	576,587
81,91*	-2.24	-7.12	-23.58	0	-0.05	0.02	0.01	-0.05	0.02	0.01	11.08	22.15	311,320	5.46	10.93	579,584
82,90*	-2.24	-8.15	-24.13	0	-0.08	-0.04	0.03	-0.08	-0.04	0.03	3.65	7.29	312,319	6.81	13.62	580,583
83,89*	-1.97	-9.07	-24.59	0	0.05	-0.16	0.03	0.05	-0.16	0.03	3.89	7.77	313,318	0.87	1.74	603,638
84,88*	-1.47	-9.80	-24.94	0	0.05	0.05	-0.04	0.05	0.05	-0.05	16.38	32.75	314,317	-0.68	-1.37	604,639
85,87*	-0.78	-10.26	-25.17	0	0.18	0.03	-0.03	0.18	0.03	-0.03	-5.52	-11.04	315,316	-1.74	-3.49	608,635
86	0.00	-10.42	-25.24	0	0.00	0.06	0.00	0.00	0.07	0.00	3.29	6.59	321,330	3.23	6.47	611,632
92,100*	2.05	-6.23	-23.02	0	0.05	0.15	0.01	0.05	0.15	0.01	5.74	11.49	322,329	3.42	6.85	615,628
93,99*	1.67	-5.52	-22.51	0	-0.09	-0.09	0.00	-0.09	-0.09	0.00	4.04	8.08	323,328	1.56	3.12	619,624
94,98*	1.17	-5.02	-22.09	0	0.00	-0.04	0.00	0.00	-0.04	-0.01	-0.93	-1.86	324,327	1.79	3.57	643,680
95,97*	0.60	-4.73	-21.83	0	0.06	0.00	0.00	0.06	0.00	-0.01	6.15	12.30	331,340	2.63	5.25	644,679
96	0.00	-4.63	-21.74	0	0.00	0.03	-0.01	0.00	0.03	-0.01	8.44	16.88	332,339	8.50	17.01	659,664
201	0.00	-7.09	-16.05	0	0.00	0.04	0.01	0.00	0.04	0.01	2.58	5.16	333,338	7.88	15.76	660,663
											4.70	9.40	334,337	1.83	3.65	683,718
											6.89	13.77	335,336	2.04	4.08	687,716
											4.97	9.94	341,350	1.37	2.73	691,712
											8.44	16.88	343,348	-1.47	-2.95	696,707
											5.50	11.00	352,359	-12.58	-25.16	699,704
											3.11	6.23	353,358	-13.03	-26.06	700,703
Total actuation (mm)															**1150.8**	

*These numbers have the opposite sign in x-direction.

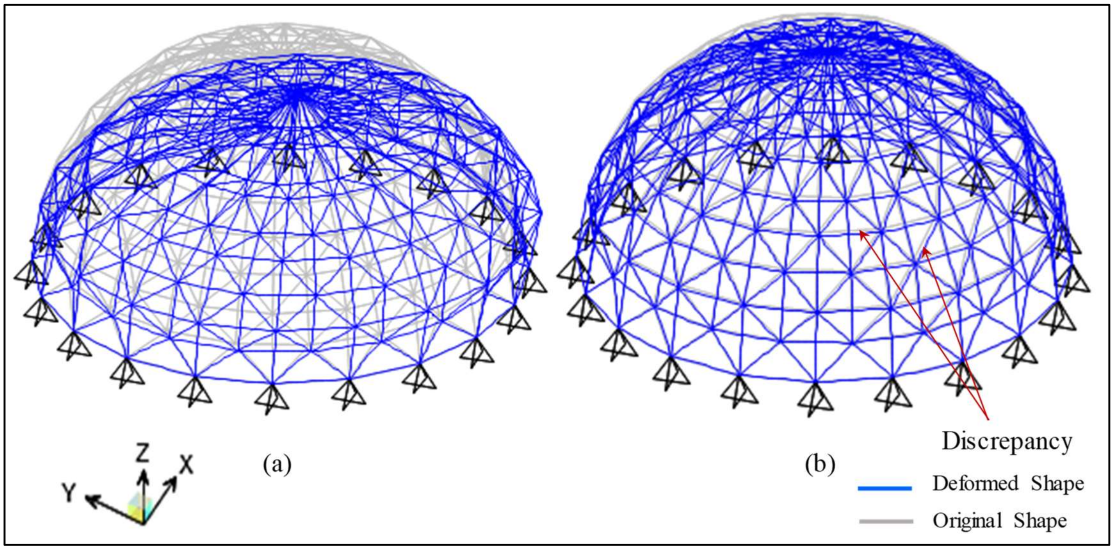

Figure 8. Double layer dome model under vertical and horizontal loading simultaneously (a) Pre-adjustment (b) Post-adjustment

Sum up of the all previous subsections, the total actuation in the last rows of all tables represents the effort (the cost in economical side) required for restoring the shape in practical, i.e., the total actuation in all cases represents the work that has to be done in all cases of loadings. The value of necessary changing in the bar length of each member, as shown in Column 12 in all tables, denotes the role of each active member in the process of the restoration of the structure.

5. CONCLUSION

In this paper, exterior nodal displacements of the double layer dome model are examined, in which its shape configuration was distorted because of the gravity and/or lateral loading by using a relatively simple and direct method via calculating the required length actuations for controlling the shape (Saeed, 2014; Saeed and Kwan, 2016). The technique was theoretically applied to the model through MATLAB program and SAP2000 software. It can be concluded that the technique of shape adjustment is very efficient for double layer dome model, and it can roughly eliminate the displacement of definite joints (exterior joints only) by simply changing the length of certain bars by $\mathbf{e_0}$ amount. In this study, three different cases corresponding to the directions of loadings were considered. In addition, it can be also concluded that in any cases of load direction, the shape controlling can be easily accomplishing. However, a very low rate of discrepancy is observed in all cases of loading, which are 0.5%, 1.8%, and 0.02%, respectively. It also founded that the position of the actuators is the crucial in order to reach the targets with the minimum amount of actuation, which in this work was totally attained as the amount of total actuations are 874.95 mm, 246.25 mm, and 1150.8 mm in Case 1, Case 2, and Case 3, respectively. These represent the effort required for restoring the shape in practical, i.e., the total actuation in all cases represents the work that has to be done in all cases of loadings. It is determined that some of the members confirm the greater role in controlling the displaced joints as they are duplicated in Case 1, Case 2, and Case 3, and they are positioned in the inner layer of the double layer dome model. Finally, it is also concluded that the applied method for such model, the type of load, and their directions are very appropriate and applicable.

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