RESEARCH ARTICLE

An Approximate Linear Analysis of Structures Using Incremental Loading of Force Method

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ABSTRACT

A relatively simple technique has been introduced in this paper. The approach is based on the linear force method with discretization of the applied loads to the subsequent steps and updating coordinates in each iteration to have a new geometrical property. The accuracy of the technique depends on the size of the increments, which affects the number of iterations. A small change in the configuration could hugely affect the displacement and internal forces in geometrically non-linear structures, which is why the current approach is vital. The proposed technique is validated with two different techniques of non-linear analysis of the structures with a very good agreement both in terms of external nodal displacements and internal bar forces.

Keywords: Force Method, Dynamic Relaxation, Geometrically Non-linear Structures, Cable Structures, Prestressed Structures

1. INTRODUCTION

In the recent years, cable net structures have been widely constructed; they are architecturally elegant structural forms. They provide clear, large spaces such as those in retractable stadium roofs, petrol stations, and tour areas. Cable structures are the most common example of geometrically non-linear structures, as they purely behave in a non-linear way. To understand the configuration of non-linear structures after loading, a technique is presented in this work which is adopted from the linear force method.

Recently, several scholars have been studying such structures, and they have introduced

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Because, direct linear methods are not accurate to show the configuration of geometrically non-linear structures after loading. In this paper, two common linear methods are mentioned: The Finite Element Method and the Force Method (FM). The former provides displacements and internal forces; however, it does not give any more detail about structural behavior. Comparatively, the latter, besides the displacement and internal force, explicitly provides the states of self-stress and possible mechanisms and, last but not least, it can identify weather a structure is behaving linearly or not for a particular case of loading (Luo and Lu, 2006). This can be simply done by P.M, in which **P** is the load vector and **M** is the possible mechanism of the structure. If P.M=0, the structure is linear for the given case of loading, otherwise the structure is geometrically non-linear for the given loading. In this study, FM has been developed to analyze the non-linear structures approximately.

The approach such simple that calls no more than basic mechanics and using numerical manipulations. Thus, the technique can even be introduced to engineering undergraduate students. This could be an easy way to introduce them to geometric non-linearity by presenting the difference in results between FM and the current technique. The layout of this paper is as follows: Section 1 is an introduction to geometrically non-linear structures. A review of previous studies has been presented in Section 2. In Section 3, development of the current technique has been explained. It is followed by an illustrative example using the proposed approach in Section 4. Section 5 contains the comparison of the current technique with other linear FM and geometrically non-linear methods in three different examples. Finally, the whole work has been concluded in Section 6.

2. LITERATURE REVIEW OR BACKGROUND

Several researchers have attempted to study geometrically non-linear structures, such as Kwan, 1998 who examined geometrically non-linear structures under loading, Coyette and Guisset, 1988 who analyzed cable network structures, and Thai and Kim, 2011 who examined pre-tensioned catenary cable element under static and dynamic loading. Similarly, the non-linear approach was also examined by Karparvarfard et al., 2015 to analyze geometrically non-linear small-scale Euler–Bernoulli beam, Stefanou et al., (1993) to analyze a saddle cable structure, and Buchholdt (1969) to study structures with finite displacements. While, different behaviors of variety of types of cable structures have been discussed in detail by Abad et al., 2013; Lewis, 2017; Naghavi Riabi and Shooshtari, 2015; Pellegrino 1990. The basics of non-linear analysis of the geometrical non-linear structures are derived from various theories such as the second strain gradient theory (Karparvarfard et al., 2015) and the conjugate gradient method (Stefanou et al., 1993). Whereas, Kwan, 2000; Luo and Lu 2006; Raju and Nagabhushanam, 2000 made a simple modification in linear methods to derive non-linear methods to analyze geometrically non-linear structures. While, an approximate method to solve non-linear structures was presented by Pellegrino, 1993. He divided the total load into two parts, the first part causes extensional nodal displacement and the second causes in extensional displacement.

In this paper, a linear technique has been modified to analyze linear and geometrically non-linear structures.

The advantage of this work is that FM is taught to civil engineering students, another benefit is that the technique is very simple. In order to prove the validity of the approach, the results of the proposed technique have been compared with two non-linear techniques presented by Lewis et al., 1984 and Kwan, 1998, and also with non-linear analysis of SAP2000 program software. Therefore, a brief introduction of these techniques and software are presented in the following subsections.

2.1. Dynamic Relaxation Method by Lewis et al., (1984)

This approach was applied for non-linear structural analysis by Lewis et al., (1984). The method is based on D'Alembert's principle, which is:

$$\boldsymbol{Q}(\boldsymbol{t}) = [\boldsymbol{M}]\boldsymbol{d}'' + [\boldsymbol{C}]\boldsymbol{d}' + [\boldsymbol{K}]\boldsymbol{d} \quad (1)$$

This technique is famous for its accuracy to analyze geometrically non-linear structures by dealing with the structure's motion from the beginning of loading till it settles down. *Q(t)* is the time dependent external load vector. The right-hand side of the formula contains three terms. The first and second parameters are $[M]d''$ and $[C]d'$ that calculate the non-linear part in which M and C are fictitious mass and damping coefficients, respectively. In addition, d' and d'' are velocity and acceleration, respectively. The last parameter is $[K]d$, which is the linear part of the equation, and K is the stiffness matrix, while $d$is the external nodal displacement.

2.2. Non-Linear Approach by Kwan (1998)

This technique is developed based on FM by Kwan, 1998, which has been used for analysis of non-linear structure, using the following equation:

$$G = \frac{EA}{L_o^3}\Delta^3 + \frac{2t_o}{L_o}\Delta \quad (2)$$

In which, G is the applied load, $EA$is the bar rigidity, L_o is the initial length of the bar, $t_o$is the pretension force in the member and Δ is the external nodal displacement.

2.3. Force Method

This approach contains three main equations, which are equilibrium $\boldsymbol{At} = \boldsymbol{p}$, compatibility $\boldsymbol{Bd} = \boldsymbol{e}$, and flexibility $\boldsymbol{Ft} = \boldsymbol{e}$ equations. where **A** is equilibrium matrix, **B** is the compatibility matrix and **F** is the flexibility matrix. The technique presented in our study is based on FM; therefore, the details of FM are discussed in Section 3.

2.4. SAP2000

In this study, the SAP2000 software is also used for non-linear analysis of structures. In order to validate the current technique proposed in this study, which is version 20.2. The program has ability to analyze linear and non-linear structures in the sense of geometric and/or material non-linearity. Thai and Kim, 2011 utilized SAP2000 to non-linear analyze of cable structures under static and dynamic loads.

3. DEVELOPMENT OF THE CURRENT TECHNIQUE

The proposed technique in this study is based on FM (Kwan, 1991; Pellegrino, 1993; Pellegrino et al., 1992; Saeed, 2014; Saeed and Kwan, 2016a, 2016b) to allow “easy access” to the contributing parameters affecting the internal forces and the external displacements. The equilibrium balance between the vector of external loads **p** and internal bar forces **t** is expressed as

$$\boldsymbol{At} = \boldsymbol{p} \quad (3)$$

where A is the equilibrium matrix, and has size$(ij - c) \times b$. The compatibility statement of the balance of internal bar elongation **e** and external nodal displacements **d** is expressed as

$$\boldsymbol{Bd} = \boldsymbol{e} \quad (4)$$

where B is the compatibility matrix, and has size$b \times (ij - c)$, and $\boldsymbol{B}^T = \boldsymbol{A}$. The flexibility relationship for a pin-jointed bar assembly has a $b \times b$ diagonal flexibility matrix F such that

$$\boldsymbol{Ft} = \boldsymbol{e} \quad (5)$$

The general solution t to the equilibrium equations is expressed as the sum of a particular solution (i.e., any vector t that satisfies Eq. (5), and one such vector is $\boldsymbol{t}_A$

obtained from $\boldsymbol{t}_A = \boldsymbol{A}^+\boldsymbol{p}$ where $\boldsymbol{A}^+$is the pseudo-inverse of A) and the complementary homogeneous solution (i.e., t satisfying$\boldsymbol{At} = \boldsymbol{0}$, which is readily provided by the nullspace(A)=S, and S is the states of self-stress). The total general solution is thus

$$\boldsymbol{t} = \boldsymbol{t}_A + \boldsymbol{S\alpha} \quad (6)$$

Substitution of Eq. (6) into Eq. (5) gives

$$\boldsymbol{e} = \boldsymbol{F}(\boldsymbol{t}_A + \boldsymbol{S\alpha}) \quad (7)$$

Compatibility is imposed by imposing bar elongations e to be orthogonal to the incompatible elongations (as found in left-nullspace(B) which is identical to nullspace(A) since$\boldsymbol{B}^T = \boldsymbol{A}$). The compatibility condition is thus$\boldsymbol{S}^T\boldsymbol{e} = \boldsymbol{0}$, i.e.,

$$\boldsymbol{S}^T\boldsymbol{e}_o + \boldsymbol{S}^T\boldsymbol{F}(\boldsymbol{t}_A + \boldsymbol{S\alpha}) = \boldsymbol{0} \quad (7)$$

from which

$$-\boldsymbol{\alpha} = (\boldsymbol{S}^T\boldsymbol{FS})^{-1}[\boldsymbol{S}^T\boldsymbol{e}_o + \boldsymbol{S}^T\boldsymbol{Ft}_A] \quad (8)$$

The expression for $\boldsymbol{\alpha}$ then reveals, by back-substitution, the structural vectors of **e** (Eq. (7)), **t** (Eq. (6)) and **d** (Eq. (4)).

The idea behind the current technique proposed in this work is the coordinate update of the structural geometry. The equilibrium **(A),** the compatibility (**B**), and the flexibility **(F)** matrices are functions of coordinates, and the change in the geometry highly affects their values. In this study, the applied load is incremental, when an increment is applied Eq. 3-8 are iteratively repeated. In each step, the output will be joint displacement and internal bar forces. While these displacements are summed to the former coordinates, the coordinates are updated and generate a new geometry. The technique is illustrated in the form of a flowchart as shown in Figure 1.

4. AN ILLUSTRATIVE EXAMPLE OF THE CURRENT TECHNIQUE

In this section, a two-bar structure has been examined using the proposed approach as shown in Figure 2. The structure has two members interconnected to each other at the mid joint, it has been supported from both ends, and $EA = 10^8 N$ for both bars. It is loaded in 10 steps in a row, in each iteration 800 N gravity load is applied to the unsupported joint. First, the structure has its original coordinates, when it is loaded, the free joint drops by 3.6477 mm; thus, new coordinates for the next step will be updated. Only the free joint will have a new position, which will be (450, -10+(−3.6477)) mm. As it is clear from Table 1 and Figure 3, when the structure is loaded with the first incremental load (i.e., the first iteration) the displacement is maximum compared to the followed iterations. In other words, in each step even for the same amount of loading the displacement plunges because of the different geometry. This is due to the fact that the members experience more stress than those in the former iteration, which increases the stiffness in its yield stress range; thus, the bar elongation decreases, and this results in less displacement.

As clearly shown in Figure 1, the flowchart explains the step-by-step algorithm of how the technique works. Initial coordinates, physical properties of the structure, the maximum load, and the number of iterations should be inputted. The necessary manipulation is performed including applying the incremented load and then the displacement is obtained until the number of iterations are reached. Finally, the loop stops and the cumulative displacement is obtained.

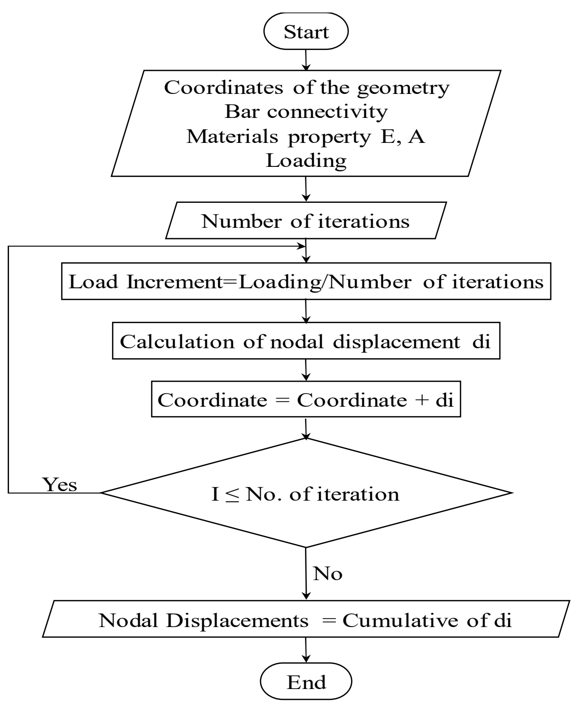

Figure 1. Flowchart of the current technique.

Figure 4 shows that the accuracy of the technique is proportional to the number of iterations. In other words, the precision of the approach is enhanced by minimizing the load increments. As it is clear from the figure the result gets closer and closer to the results of Dynamic Relaxation when the number of iterations is increased. In this example with applying only 10 load increments, the results have a very good agreement with non-linear technique of analysis (Dynamic Relaxation). Numerically speaking, when FM is used (*i.e.* number of iterations is one) the displacement of the free joint is −36.466 mm as shown in the labeled line of Figure 4. While, for two iterations, the displacement rested at −20.5375 mm and so on when the number of iterations is 10 the displacement is −13.5421 mm which is very close to the exact geometrically non-linear analysis method Dynamic Relaxation which gives -11.1168 mm.

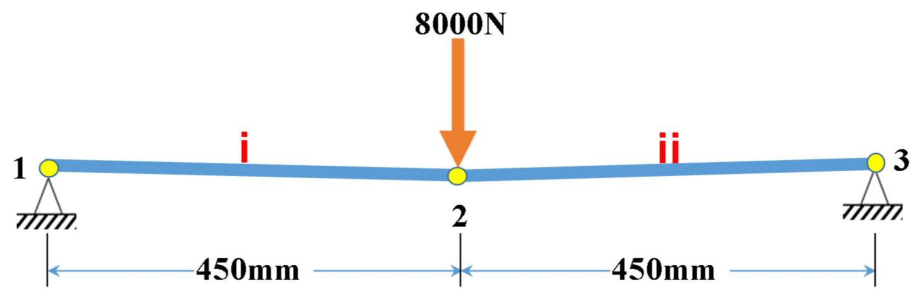

Figure 2. Structure 1, the two-bar geometrically non-linear structure. J1 and J3 are in the same level but J2 is 10 mm bellow J1

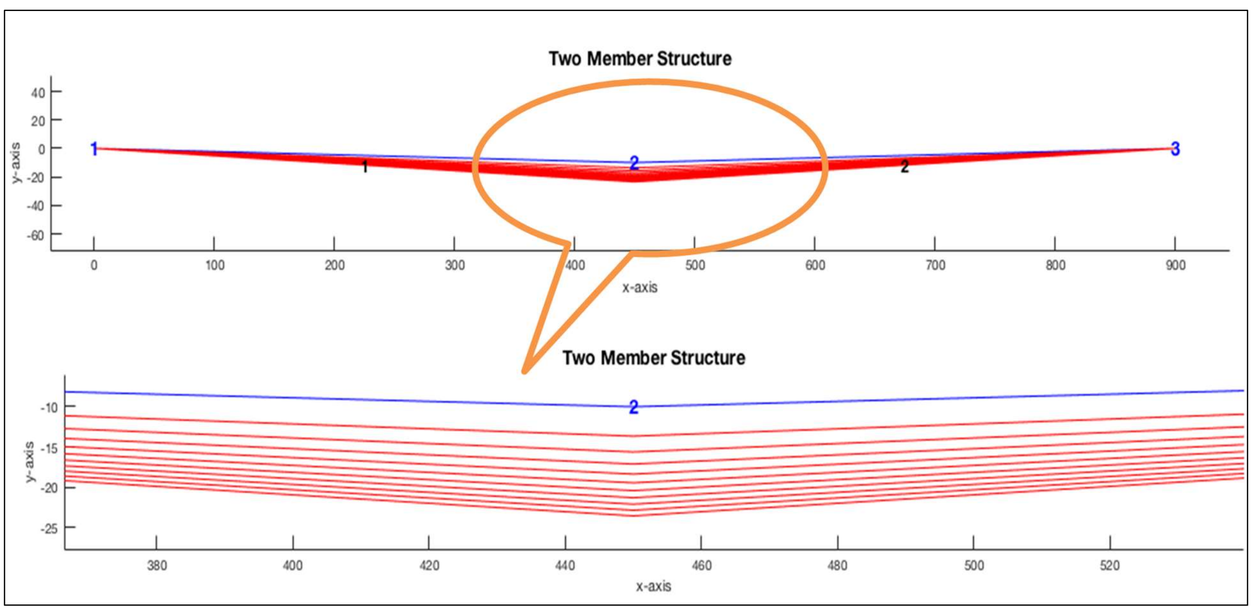

Figure 3. Structure 1 configuration in 10 iterations

Table 1: The components that are used in the project

1	2	3	4	5	6	7	8	9
Iterations	Coordinates(mm)						Load (N)	Vertical displacement of J2 (mm)
	J1		J2		J3			
	x	y	x	y	x	y		
1	0	0	450	−10	900	0	−800	−3.6477
2	0	0	450	−13.6477	900	0	−800	−1.9596
3	0	0	450	−15.6073	900	0	−800	−1.4991

4	0	0	450	−17.1064	900	0	−800	−1.2483
5	0	0	450	−18.3547	900	0	−800	−1.0846
6	0	0	450	−19.4394	900	0	−800	−0.9673
7	0	0	450	−20.4066	900	0	−800	−0.8780
8	0	0	450	−21.2846	900	0	−800	−0.8073
9	0	0	450	−22.0919	900	0	−800	−0.7495
10	0	0	450	−22.8415	900	0	−800	−0.7013
			Total				−8000	−13.5428

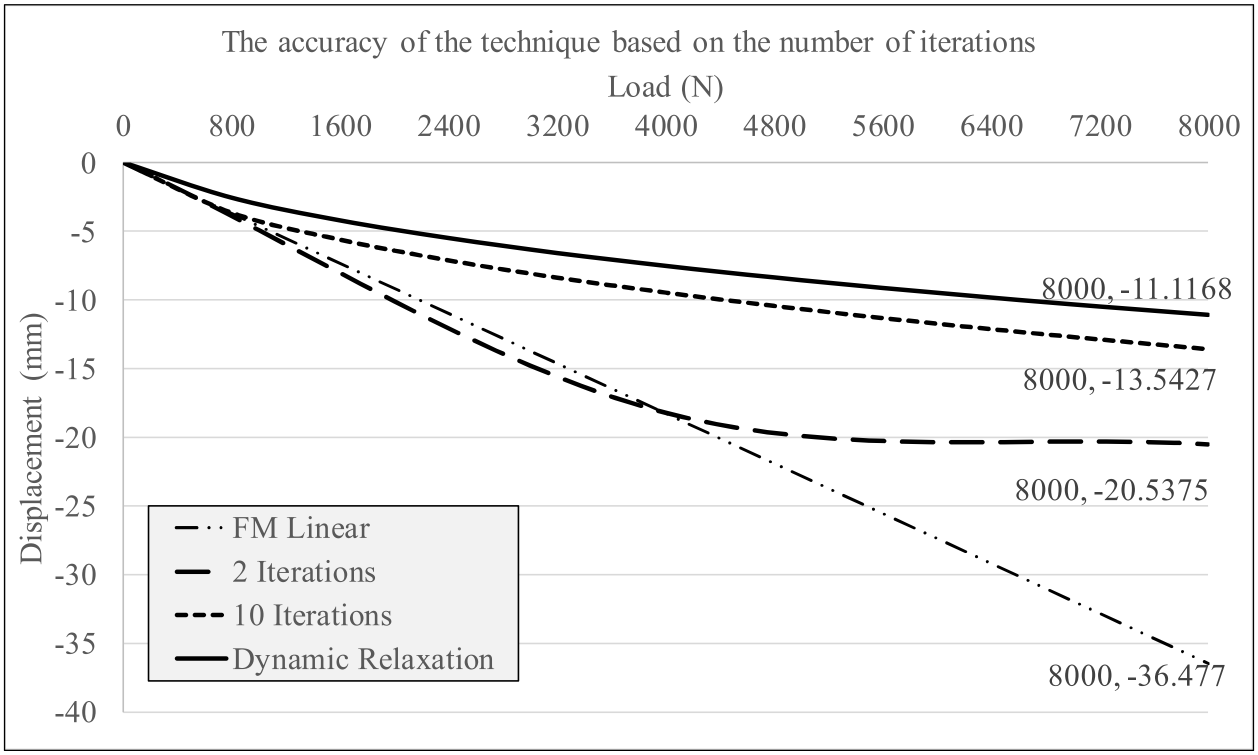

Figure 4. Enhancement of the accuracy of the technique is proportional with the number of iterations

5. RESULTS and DISCUSSION

In this section a detailed comparison of the results of the current technique and the quoted methods for the structure examined in Section 4 and two other structures have been presented.

5.1. Structure 1

The properties of this structure have been detailed in Section 4 and the structure's geometry is shown in Figure 2. It can be clearly seen from Table 2 that there is a substantial discrepancy of the vertical displacement of J2 between FM and the approved techniques, the dissimilarity is about 228%. Whereas the difference of the results of the current approach with the quoted methods is around 16%. It can be said that the geometry of the geometrically non-linear structures is highly significant. Direct linear methods are deficient for such structures that is why this new technique is essential. In this method the applied load is discretized to number of iterations, this leads to reduction of sensitivity of the structure to the load, because the load is applied incrementally. The results of the internal loadings also have good relations with the non-linear techniques as compared to the results from linear FM.

Table 2: Comparison of the Current Technique with the Other Methods in Terms of Displacement and Internal Force for Structure 1

		FM	Current Study	Kwan	Lewis	SAP2000
Vertical displacement(mm)	Joint 2	−36.4770	−12.9887	−11.1120	−11.1168	−11.1169
Internal Force (N)	Bar 1	180040	104580	85303	85334	85320
	Bar 2	180040	104580	85303	85334	85320

5.2. Structure 2

In this example, a three-dimensional structure is studied as shown in Figure 5, simply two other members were connected to the mid joint of the structure 1 from out of plain direction and the load has been doubled as presented in the figure. This has been performed to test the technique with three-dimension structures and to see whether the results match with the results of structure 1. The properties are unchanged; the unsupported joint is subjected to 16,000 *N* downward. As one sees the results from Table 3, there is no amendment in the sense of the amount of displacement and internal forces. It can be said that the technique is also applicable to analyze three-dimensional structures.

5.3. Structure 3

An eight-bar truss see Figure 5 has been examined using FM, the current study and the two quoted non-linear approaches. This structure is predicted to behave linearly because it is made out of steel; thus, it is not a cable structure, 1400 kN downward has been applied to the far joint to get a noticeable displacement. This is done to see the precision of the current study to analyze linear structures. The structure has five joints and eight bars with EA= 200,000* [650 750 500 500 750 500 650 650], respectively based on the bars' number. In this example number of iterations is 100. Table 4 clearly shows that the results of the current technique are closer to the quoted exact methods than the FM method. The discrepancy between FM and the approved non-linear methods is just above 3.5% whereas the dissimilarity between the current technique and the non-linear methods is just under 1.7%.

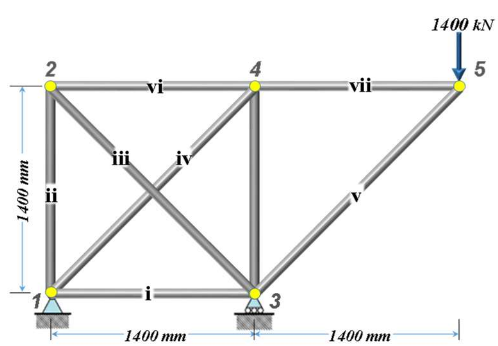

Figure 5. Structure 3, a linear and determinate truss with eight members

Table 3: Vertical Displacements of the Loaded Joint of Structure 3, Comparing the Current Technique with The Quoted Methods.

		FM	Current study	Lewis	Kwan	SAP2000
Vertical displacement(mm)	Joint 5	−112.3045	−114.4245	−116.3915	−115.7500	−115.5500

6. CONCLUSION

A relatively simple and approximate method is proposed in this study, which is accurate to solve linear structures and effectual to analyze both two and three-dimensional geometrically non-linear structures. However, the degree of accuracy depends on the number of iterations and the precision is proportional to the number of iterations. Besides the lack of complications, the proposed technique has been proved to be very simple and efficient compared to ordinary linear methods given the results of the proposed method are comparatively close to the established non-linear techniques in its accuracy of solution.

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